Journal of Experimental Child Psychology 166 (2018) 116–133

Contents lists available at ScienceDirect

Journal of Experimental Child Psychology

journal homepage: www.elsevier.com/locate/jecp

What counts in preschool number knowledge? A Bayes factor analytic approach toward theoretical model development

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ARTICLE INFO

Article history:
Received 29 November 2016
Revised 29 May 2017

Keywords:
Early symbolic number knowledge
Counting
Approximate number system
Numerical cognition
Bayes factor
Executive function

ABSTRACT

Preschool children vary tremendously in their numerical knowledge, and these individual differences strongly predict later mathematics achievement. To better understand the sources of these individual differences, we measured a variety of cognitive and linguistic abilities motivated by previous literature to be important and then analyzed which combination of these variables best explained individual differences in actual number knowledge. Through various data-driven Bayesian model comparison and selection strategies on competing multiple regression models, our analyses identified five variables of unique importance to explaining individual differences in preschool children's symbolic number knowledge: knowledge of the count list, nonverbal approximate numerical ability, working memory, executive conflict processing, and knowledge of letters and words. Furthermore, our analyses revealed that knowledge of the count list, likely a proxy for explicit practice or experience with numbers, and nonverbal approximate numerical ability were much more important to explaining individual differences in number knowledge than general cognitive and language abilities. These findings suggest that children use a diverse set of number-specific, general cognitive, and language abilities to learn about symbolic numbers, but the contribution of number-specific abilities may overshadow that of more general cognitive abilities in the learning process.

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http://dx.doi.org/10.1016/j.jecp.2017.07.016

Introduction

Children growing up in industrialized societies begin to acquire a symbolic number system before entering elementary school (Carey, 2009; Gelman & Gallistel, 1978; Wynn, 1990, 1992). This process starts at around 2 years of age, when children start reciting the count list without necessarily understanding the words they are saying, and develops for several more years as children come to understand that each number word in the list represents a unique cardinal value, that the count list can be employed in a counting routine to determine cardinality of any given set of items, and that every number on the count list has a unique successor (Cheung, Rubenson, & Barner, 2017; Fuson, 1988; Gelman & Gallistel, 1978; Le Corre & Carey, 2007; Wynn, 1992). This early preschool symbolic number knowledge appears to lay a foundation for higher mathematics, as it is highly predictive of later mathematical achievement even after controlling for other general cognitive and demographic factors (Jordan, Kaplan, Ramineni, & Locuniak, 2009; Nguyen et al., 2016; vanMarle, Chu, Li, & Geary, 2014). Given its foundational role, it becomes important to know what cognitive and language abilities give rise to individual differences in preschool children's early number knowledge.

Foundations of mathematical thought

A majority of empirical and theoretical work to date has focused on the foundations of later developing mathematical thought. This work suggests that mathematical thinking is generally subserved by at least three components: number-specific, language, and general cognitive abilities (e.g., Cirino, 2011; Dehaene, Piazza, Pinel, & Cohen, 2003; LeFevre et al., 2010). However, the contribution of each component varies with the particular mathematical skill or outcome of interest and across development (e.g., Cirino, 2011; LeFevre et al., 2010; Sowinski et al., 2015). To our knowledge, such theories have yet to be tested at the earliest stages of symbolic numerical concept development in preschoolers. Thus, although these existing theories of the foundations of mathematics provide a general framework for thinking about the types of cognitive and linguistic factors that may be important to thinking about numbers, their particular contributions to initial symbolic number knowledge before children enter school remain unclear.

Sources of individual differences in early number knowledge

Other studies have focused on the foundations of earlier symbolic number knowledge but in a more piecemeal fashion (Abreu-Mendoza, Soto-Alba, & Arias-Trejo, 2013; Brannon & Van de Walle, 2001; Kroesbergen, Van Luit, Van Lieshout, Van Loosbroek, & van de Rijt, 2009; Mussolin, Nys, Content, & Leybaert, 2014; Mussolin, Nys, Leybaert, & Content, 2012; Negen & Sarnecka, 2012; Shusterman, Slusser, Halberda, & Odic, 2016; vanMarle et al., 2014, 2016; Wagner & Johnson, 2011). That is, particular studies have focused on relationships between a particular cognitive or language ability and early symbolic number knowledge.

From this work, knowledge of the count list has been proposed as foundational for conceptual gains in early symbolic number system understanding (e.g., Carey, 2009; Gelman & Gallistel, 1978; Le Corre, Van de Walle, Brannon, & Carey, 2006; Wynn, 1990, 1992). Children start to acquire a symbolic number system by memorizing the number words and their fixed order (i.e., the count list) without necessarily understanding the numerical meanings of the words or knowing how to employ them to count precisely (Carey, 2009; Fuson, 1988; Wynn, 1990, 1992). Furthermore, even after cardinal meanings of the first number words are acquired, knowledge of the count list continues to precede deeper understanding of symbolic number (Carey, 2009; Davidson, Eng, & Barner, 2012; Fuson, 1988; Le Corre et al., 2006). Therefore, although count list knowledge may be a necessary developmental component, it is certainly not sufficient for either conceptual or procedural mastery of the symbolic number system (Carey, 2009; Fuson, 1988; Le Corre et al., 2006). Given its proposed importance, it is not surprising that knowledge of the count list has been shown to correlate with deeper number knowledge (Davidson et al., 2012; Mussolin et al., 2012, 2014; vanMarle et al., 2014). However, the relative importance of count list knowledge compared with other number-specific and domain-general cognitive abilities remains unclear.

It has also been proposed that children use nonverbal, approximate numerical intuitions in acquiring symbolic number knowledge (Dehaene, 1997; Gallistel & Gelman, 1992, 2000; but see Le Corre & Carey, 2007). Several decades of research now show that even very young children and infants are able to reason about and act on the approximate numerosity of sets of objects nonverbally (see Anderson & Cordes, 2013, and Feigenson, Dehaene, & Spelke, 2004, for reviews). This ability has been referred to as the approximate number system (ANS) (Dehaene, 1997; Feigenson et al., 2004; Gallistel & Gelman, 2000). It has been hypothesized that the ANS forms an important conceptual foundation for understanding symbolic number words and, thus, should be associated with early number knowledge (Dehaene, 1997; Gallistel & Gelman, 1992, 2000; vanMarle et al., 2016; Wagner & Johnson, 2011). Some studies have shown an association between individual differences in nonsymbolic approximate numerical abilities, typically measured as accuracy in a nonsymbolic numerical comparison task (e.g., comparing sets of objects), and individual differences in early number knowledge (Abreu-Mendoza et al., 2013; Mussolin et al., 2012; Mussolin et al., 2014; Shusterman et al., 2016; vanMarle et al., 2014, 2016; Wagner & Johnson, 2011). However, the relation between nonsymbolic approximate numerical abilities and symbolic number knowledge is controversial, with a few other studies reporting no relation between the two (Huntley-Fenner & Cannon, 2000; Negen & Sarnecka, 2015).

Inconsistency in the findings of the relations between the ANS and early number knowledge parallels the inconsistency in the findings of the relations between the ANS and later developing mathematics in older children and adults. That is, a number of studies show a relationship between the ANS and mathematics achievement, whereas other studies do not (see Chen & Li, 2014, and De Smedt, Noël, Gilmore, & Ansari, 2013, for reviews). One predominant explanation for the mixed results is that the relation between the ANS and mathematics ability may be confounded by general cognitive factors (e.g., Fuhs & McNeil, 2013; Gilmore et al., 2013; but see Keller & Libertus, 2015). In many contexts, comparing sets of items on number requires one to actively inhibit conflicting information or responses to other non-numerical properties such as the individual item size or total area of the objects, and such executive processes also highly correlate with children's early mathematical abilities (Fuhs & McNeil, 2013; Gilmore et al., 2013). Controlling for executive processes has been show to severely reduce or even eliminate statistical relations between mathematics achievement and approximate numerical abilities in several studies (e.g., Fuhs & McNeil, 2013; Gilmore et al., 2013; but see Keller & Libertus, 2015).

Like the case of later mathematics achievement, executive functions may also confound the relationship between the ANS and earlier symbolic number knowledge. Most previous work on the relation between the ANS and early number knowledge has not controlled for general cognitive abilities (Abreu-Mendoza et al., 2013; Rousselle, Palmers, & Noël, 2004; Shusterman et al., 2016; Wagner & Johnson, 2011). Although the few studies that have attempted to control for general cognitive abilities showed that the relation between the ANS and early understanding of cardinality holds (Mussolin et al., 2014; vanMarle et al., 2014, 2016), further work is needed to more deeply understand this relationship.

Finally, many have hypothesized a specialized role of language in learning symbolic numbers (see Barner, 2017; Carey, 2009, and Spelke, 2011, for reviews). As evidence of this, language abilities, such as young children's symbolic knowledge of letters and words, as well as their general receptive vocabulary have been found to correlate with early mathematical abilities, including symbolic number knowledge (Chu, vanMarle, & Geary, 2015; Fuhs & McNeil, 2013; Mussolin et al., 2012; Negen & Sarnecka, 2012; vanMarle et al., 2014, 2016). Furthermore, LeFevre et al. (2010) showed that language-related abilities were related to seven different early mathematics outcomes measured in children, including early symbolic number knowledge, and that at least for some mathematics outcomes language-related abilities showed stronger associations than the number-specific and general cognitive abilities.

In sum, there is empirical support for the contribution of number-specific, language, and general cognitive abilities to early symbolic number knowledge. However, the unique contribution and relative importance of each to early preschool symbolic number knowledge acquisition remains understudied and unclear.

The current study

Here we used an individual differences approach to better understand the foundations of early symbolic number knowledge. Specifically, we assessed the extent to which individual differences in verbal and nonverbal numerical abilities, general cognitive abilities, and language abilities are related to individual differences in children's early symbolic number knowledge. We followed a similar logic as used in previous cross-sectional studies of mathematics achievement with older children (e.g., Cirino, 2011; Fuchs, Geary, Compton, Fuchs, & Hamlett, 2010; Libertus, Feigenson, & Halberda, 2011; Sowinski et al., 2015), reasoning that if a given cognitive or language ability uniquely contributes to number development in preschoolers, then individual differences in that ability would likely be associated with individual differences in number knowledge even after accounting for other associated abilities.

We focused on a cross-sectional sample of 3- and 4-year-old children in the midst of acquiring a symbolic number system because this has been documented as a unique time point in development where individual children are likely to maximally differ in symbolic number knowledge (Le Corre et al., 2006; Sarnecka & Lee, 2009; Wynn, 1990, 1992). Before this age, most children have little if any symbolic number knowledge, and it is difficult to assess cognitive and language abilities (e.g., Negen & Sarnecka, 2015; Odic, Paul, Hunter, Lidz, & Halberda, 2013). Shortly after this age, typically developing children begin to perform closer to ceiling on early number knowledge assessment tasks (e.g., Chu, vanMarle, & Geary, 2013), and remaining individual differences become further confounded with formal instruction in school.

Given that the contribution and relative importance of hypothesized variables to early number knowledge are currently unknown, we took a data-driven approach toward theoretical model selection. We started with number-specific, language, and general cognitive variables thought to be important to early number knowledge and computed multiple regression models of all possible unique combinations. We then employed a Bayesian analytic approach to alternative model selection (Rouder & Morey, 2012). This approach provides both a transparent quantitative metric to directly compare alternative models (i.e., identifying the best combination of cognitive abilities that explain variance in symbolic number knowledge) and allows one to evaluate the relative explanatory power of specific variables in each model (i.e., ranking variables by importance to model). The core of this approach is to compute a Bayes factor, which is a ratio of the probability of the data given one model (e.g., Model M_A) relative to another model (e.g., Model M_B) (i.e., $P(Y|M_A)/P(Y|M_B)$, where P denotes probability and Y denotes the data) (see Rouder & Morey, 2012, for details on computing Bayes factors). For instance, if the comparison of two hypothetical models, A and B, yielded a Bayes factor of 2 (denoted $B_{A\_B} = 2$), then this would indicate that the data are twice as probable under Model A (M_A) compared with Model B (M_B). Therefore, the magnitude of the Bayes factor indicates the extent of evidence in favor of one model compared with another. The larger the Bayes factor, the more evidence in favor of the first given model (e.g., Model A); the closer the Bayes factor to 1, the more similar the two models in their explanatory power of the data. Alternative models can then be directly compared and contrasted by Bayes factors.

Because alternative models are composed by the systematic inclusion of some variables and the exclusion of others, the Bayes factor also provides a measure of the relative explanatory power lost or gained by removing or adding specific variables (Rouder & Morey, 2012). Thus, computing and comparing the Bayes factors among many different alternative models can also be informative as to the relative importance of the variables uniquely contributing to the model (Moore, vanMarle, & Geary, 2016; Mou et al., 2016; Rouder & Morey, 2012). Therefore, compared with traditional model identification and comparison methods (e.g., stepwise regression analysis), the Bayes factor approach provides a convenient option to both identify the best combination of variables that uniquely explain individual differences in early number knowledge and quantify their relative importance. Moreover, the evaluations on the models and the variables are carried out on a transparent quantitative metric (i.e., the magnitude of Bayes factors). Thus, the evaluation results can be interpreted straightforwardly (e.g., Bayes factor = 5 indicates that data are 5 times as probable under one model relative to the other), which is more difficult in traditional model selection approaches with significance tests (Rouder & Morey, 2012).

Method

Participants

A total of 131 preschool-aged children from the Midwest region of the United States (60 boys) participated in our study and had complete data for all of the tasks of interest (M_{age} = 3 years 10 months 25 days, *SD* = 52 days, range = 3 years 7 months 19 days to 4 years 3 months 18 days). An additional 17 children (8 boys) were eliminated for incomplete data records. For children in the final sample, parents reported that 54.2% of the children had attended or were currently attending preschool. Parents of these child participants had a modal education level of a bachelor's degree (39.5% of 124 parents who reported education level) and a modal yearly household income of $60,000 to $80,000 (22.3% of 121 parents who reported household income).[1]

Written consent for children's participation was obtained from a parent or legal guardian. Children received a small gift for their participation, and parents were reimbursed for travel expenses. This study was conducted under the approval of the University of Illinois at Urbana–Champaign institutional review board.

Materials and procedure

Data for this study were collected on the first visit of a 2-week computer-based numerical training study. All of the tasks were administered in a fixed order as described below.

Count list knowledge

Count list knowledge (count list) in the current study is taken as a proxy for number-specific experience (see Davidson et al., 2012, for a similar rationale). To assess this, children were asked to recite the count list, counting as high as possible starting with "one," with a stop rule at "twenty-five." The highest number word recited correctly in sequence was recorded as the score for the task (with 25 being the maximum score children could receive).

Symbolic number knowledge

Early symbolic number knowledge (number knowledge) in the current study refers to children's conceptual understanding of the spoken number words and counting. To assess this, we measured children's ability to produce and label exact cardinal values 1–8. More specifically, number knowledge was assessed using the composite score (average accuracy) from modified versions of two well-established tasks administered on a computer: the give-a-number (*Give-N*) task (Wynn, 1990, 1992) and the what's-on-this-card (*WOC*) task (or how-many task) (Gelman, 1993; Le Corre et al., 2006).

[1] Further demographic descriptive statistics include the following. The reports on the parental educational level showed that 13.7% of parents had obtained professional degrees (e.g., M.D., Ph.D.; *n* = 17), 27.4% had obtained master's-level degrees (e.g., M.A., M.S.; *n* = 34), 39.5% had obtained bachelor's-level degrees (e.g., B.A., B.S.; *n* = 49), 11.3% had obtained associate's degrees (*n* = 14), 7.3% had high school diplomas (*n* = 9), and 0.8% had less than a high school diploma (*n* = 1). The educational levels were coded on a 6-point scale (professional degree = 1 and less than a high school diploma = 6). The yearly household income reports showed that 0% of the families had incomes less than $9,999 (*n* = 0), 1.7% had incomes from $10,000 to $19,999 (*n* = 2), 5.0% had incomes from $20,000 to $39,999 (*n* = 6), 16.5% had incomes from $40,000 to $59,999 (*n* = 20), 22.3% had incomes from $60,000 to $79,999 (*n* = 27), 13.2% had incomes from $80,000 to $99,999 (*n* = 16), 14.0% had incomes from $100,000 to $119,999 (*n* = 17), 9.9% had incomes from $120,000 to $139,999 (*n* = 12), 5.8% had incomes from $140,000 to $159,999 (*n* = 7), 3.3% had incomes from $160,000 to $169,999 (*n* = 4), 2.5% had incomes from $170,000 to $179,999 (*n* = 3), and 5.8% had incomes of $180,000 or more (*n* = 7). These levels were coded on a 12-point scale (<$9999 = 1 and $180,000 or more = 12). No strong correlations were observed between children's symbolic number knowledge and the demographic factors, including parental educational levels, household income, children's preschool entry, and gender. Because these correlations were low (*r*s < .192, Bayes factors < .83, favoring no correlations between variables), these factors were not included in formal data analyses.

Although both of these tasks typically have been used to measure children's early conceptual knowledge of numbers, they tap into somewhat different aspects of number understanding (see Gelman, 1993, Le Corre et al., 2006, and Wynn, 1990, 1992, for discussions); the give-a-number task requires children to produce a requested cardinality, and the what's-on-this-card task (or how-many task) asks children to identify the cardinal value of a set of items (Gelman, 1993; Wynn, 1990, 1992). Thus, we combined scores from the two tasks as a composite measure of symbolic number word understanding in the current study (see Hyde, Simon, Berteletti, & Mou, in press for similar implementation).

Give-a-number. In the Give-N task, children were presented with 10 items (e.g., butterflies) located in a row at the top of the computer screen and heard the computer request a given number (e.g., "five"). Children were asked to give the number of items as requested by pressing the spacebar, with each press moving one item from the top to the center of the screen. Children were encouraged to count aloud when pressing the spacebar or to count the items moved down. In the first test block, children were given a practice trial at the beginning of the block in which they were asked to give one item. Only for this practice trial did children receive verbal feedback from the experimenter. After this trial, children received seven test numbers from "two" to "eight" that were requested in a random order. Then, children received two more test blocks, each having eight test numbers from "one" to "eight" presented in a random order. Each block had a different type of item (butterflies, fish, or birds). Children were allowed to restart a trial if they thought they had given a wrong number and wanted to correct it regardless of whether the answer was really wrong. The total percentage of correct responses (out of 24 trials) was used as the score for the test.

It is important to note that some studies use knower level, or the highest number the child shows evidence of understanding, as the dependent variable on the Give-N task (e.g., Le Corre et al., 2006; Lee & Sarnecka, 2010; Piantadosi, Jara-Ettinger, & Gibon, 2014; Wynn, 1992). Operational definitions of knower level vary tremendously by task context and research group, making it challenging to definitively compute and compare across studies. Instead, we chose percentage correct because it can be objectively computed and directly compared across studies.

What's-on-this-card. In the WOC task, children were presented with pictures containing 1–8 items on the computer screen and were asked, "How many X [where X could be apples, butterflies, or fish]? Can you count them?" Children were encouraged to use their fingers to count the items one by one. The last number word children produced in the counting process was recorded as the answer, and the experimenter did not ask the "How many?" question again after the counting process. The experimenter recorded that number (e.g., "two") by pressing the corresponding number key on the keyboard. The task started with a practice trial with 1 item, and children received feedback from the experimenter for this trial (e.g., "There is one apple"). After this practice trial, sets between 1 and 8 items were presented in a random order in a test block. Three blocks were given (no practice trial in the latter two blocks), each containing a different type of item (apples, butterflies, or fish). For the same reason as in the Give-N task, a total percentage of correct responses (out of 25 trials, including performance in the first 1-item trial) was used as the score for the WOC task.

Approximate numerical comparison

This task (*approx. number*) assessed children's ability to make nonverbal approximate numerical comparisons by requiring them to point to the more numerous set of items (e.g., Halberda, Mazzocco, & Feigenson, 2008). In this task, children were shown a display containing two sets of dots side by side and were asked to point to the set with more dots. One set had red dots and the other had blue dots, and side of presentation was counterbalanced. For the first 10 trials, children received feedback in the form of "smiley" and "frowny" faces to indicate correct and incorrect answers, respectively. No feedback was given for the remaining 54 test trials. Sets were presented on the screen until children gave a response. Seven ratios (1:2, 2:3, 3:4, 4:5, 13:16, 7:8, and 8:9) were used to create sets, and numbers ranged from 8 to 32, with one of the sets always being 16 (i.e., reference). Test trials were presented in a random order, with the larger set appearing equally often on the left and right sides of the screen. Although randomly presented, half of trials contained sets equated on individual dot size, where the number of dots was positively correlated with total area of the dots in the array or was congruent with non-numerical spatial properties (i.e., congruent trials). The other half of trials contained sets equated on total area, where the number of dots was inversely related to size of the dots or was incongruent with non-numerical spatial properties (i.e., incongruent trials). This was done so that non-numerical features of the sets were not predictive of the correct answers in the experiment as a whole. A total percentage correct (out of 64 trials) was used as the main score for the test. Since incongruent trials have been hypothesized to be more demanding for general cognitive abilities (e.g., executive functions) than the congruent trials, we also computed percentage correct on congruent trials (32 trials) and incongruent trials (32 trials) and included both in some supplementary analyses to determine whether trials demanding more or less general cognitive abilities have similar or different contributions compared with early number knowledge.

It is important to note that all comparison trials, including the 10 trials with feedback, were included to maximize the number of trials used to compute individual participant scores. However, it is not likely that the inclusion of the feedback trials would influence the relations of the individual differences between the numerical comparison performance and symbolic number knowledge given that all children received the same number of trials with feedback at the beginning of the task.

Language abilities

Peabody Picture Vocabulary Test IV. Children's receptive vocabulary was assessed in this picture-based vocabulary task (*recept. vocab.*) (Dunn & Dunn, 2007). In this task, children heard a word and were required to point the corresponding picture among four potential choices. The standardized protocol for test administration was followed. Children started the assessment from a level corresponding to their age (e.g., in the current study, children started from the level for age 3 or 4 years). To set the basal level, the criterion requires that 11 of 12 items must be correct in the starting level; otherwise, the previous level, corresponding to a younger age, must be administered. Children move to the next level if they have at least 5 of 12 correct items. The test is stopped when children make 8 or more mistakes on a given level. Raw scores were recorded and also converted to standardized scores according to age norms.

Woodcock–Johnson III Letter–Word Identification subtest. This test (*letter id.*) was used to measure early reading achievement, particularly symbolic knowledge of letters and words (Woodcock, McGrew, & Mather, 2001). In this task, children were shown cards with letters or words. More specifically, the test started by requiring children to identify letters among potential choices (e.g., "point to E"), and progressively children were asked to name the letters or words (e.g., "What is the name of this letter"). All children started with the first item and stopped when six consecutive mistakes were made. Raw scores were recorded as the number of correct answers and were converted to standardized scores according to age norms.

General cognitive abilities

Three different tasks were administered to assess general cognitive abilities: working memory, conflicting information processing, and response inhibition.

N-back working memory task. This task (*working mem.*) assessed visual working memory for objects using a modified version of the classic N-back memory task (e.g., Kirchner, 1958). In this task, children were shown pictures of objects (e.g., table, coat, umbrella) sequentially presented on the screen and were asked to indicate whether a picture was the "same" as or "different" from all of the pictures previously seen. Pictures were repeated either immediately (1-back) or after one novel picture (2-back). Children were required to press a keyboard button when they thought the same picture was repeated. The experimenter and children together completed the first sequence of pictures until the first repeated picture was shown. During the first sequence, the experimenter said aloud "different, different . . ." and said "same! So we press the button!" when the first repeated picture appeared. After this, children completed the remaining trials under the supervision of the experimenter without receiving feedback from the experimenter. Pictures were shown for 2 s. and were separated by 500 ms. Children were presented with 60 pictures, 36 of which were nonrepeated and 12 of which were repeated. The 12 repeated pictures were evenly split between 1-back and 2-back conditions. The computer sounded with a positive chime (i.e., high pitch) as feedback for correct responses and no feedback was generated by the computer for errors or misses. Performance in this task was scored as a percentage of correct hits out of 12 test trials.

Spatial conflict processing task. This task (*conflict*) assessed executive processing of conflicting spatial information using a modified version of the Simon task (Willoughby, Wirth, Blair, & Family Life Project Investigators, 2012). The task was composed of three short blocks where children judged the direction of arrows presented on the computer screen by pressing the corresponding keys on the keyboard (pressing "F" for left-pointing arrows and pressing "J" for right-pointing arrows). However, the locations of the arrows on the screen changed in the three blocks. In the first block of 8 trials, arrows were presented in the center of the screen and pointed to the left or right alternately across trials. In the second block of 14 trials, arrows were presented on the left or right side of the screen and always pointed congruently with the location in which they were presented (e.g., an arrow that was presented on the left of the screen always pointed to left). In the final test block of 20 trials, arrows were presented on the left or right side but pointed either congruently or incongruently with the location they were presented. Congruent trials were the same as in the second block, and incongruent trials presented an arrow on one side that pointed to the opposite direction (e.g., an arrow that was presented on the left side of the screen pointed to the right). In the incongruent trials, children received conflicting spatial information from the location and the pointing direction of the arrow. Children needed to resolve the conflicting information by applying the instructions of the task to focus on where the arrow pointed. Positive (higher pitched) or negative (lower pitched) chimes were given as feedback for correct and incorrect responses, respectively, for all trials. The percentage correct in each block was recorded. To focus on children's ability to process conflicting information, only performance from the third test block was included in the analyses.

Go/No-go response inhibition task. This task (*resp. inhibition*) was used to assess response inhibition using a modified version of the classic go/no-go task (Durston et al., 2002; Willoughby et al., 2012). In this task, children saw sequentially presented pictures of cartoon animals on the computer screen. Children were asked to press a button on the keyboard for every animal that appeared on the screen (i.e., the *go* trials) except when a snake appeared. In this case, children were instructed to not press the key when seeing the snake (i.e., the *no-go* trials). Pictures were presented for 2 s. and were separated by 250-ms. blank screen. There were 60 pictures in total, and 15 of them were the snake pictures (i.e., 15 no-go trials). Two snake pictures were separated by 1, 3, or 5 pictures of other animals. Positive (higher pitched) and negative (lower pitched) chimes were given as feedback for correct and incorrect responses, respectively. Because the focus of the task was on assessing children's response inhibition, only the percentage of correct responses on the no-go trials (out of 15) was used as the score for this task.

Analysis

We used a Bayes factor analytic approach (Liang, Paulo, Molina, Clyde, & Berger, 2008; Rouder & Morey, 2012) to identify the best combination of the variables of interest to early symbolic number knowledge as well as the relative importance of these variables. Given that all variables in our design have been hypothesized to be important but their actual contribution and relative importance to early symbolic number knowledge is unknown, we took a data-driven approach toward Bayes factor model selection. To do this, we first automatically computed the regression models for all possible combinations of eight variables of interest included in our study (256 total models), including seven variables derived from performance on our assessment tasks (count list knowledge, approximate numerical comparison accuracy, working memory, response inhibition, conflict processing, receptive vocabulary, and letter–word knowledge) and age. We then computed the Bayes factor between each of the models and the null model (i.e., a model including only the intercept, with each Bayes factor in this analysis denoted as $B_{\mathrm{m\_null}}$, where the subscript "m" represents a particular model tested and "null" represents the null model; see the fourth column in Table 1). All Bayes factors for each model relative to the null model (i.e., $B_{\mathrm{m\_null}}$) were then ordered based on their values. As shown in Table 1, the very large values

Table 1
Bayes factor analyses for assessment of fitting models.

Model		Number of variables	Bayes factors	
Name	Variables		$B_{\mathrm{m-null}}$	$B_{\mathrm{top\_m}}$
Top-performing model (M_{top})	count list + letter id. + approx. number + working mem. + conflict processing	5	1.31×10^{16}	1
M_2	count list + letter id. + approx. number + working mem. + conflict processing + resp. inhibition	6	1.26×10^{16}	1.04
M_3	count list + letter id. + approx. number + working mem. + conflict processing + resp. inhibition + recept. vocab.	7	5.12×10^{15}	2.56
M_4	count list + letter id. + approx. number + working mem. + conflict processing + recept. vocab.	6	4.53×10^{15}	2.89
M_5	count list + letter id. + approx. number + working mem. + resp. inhibition	5	4.16×10^{15}	3.15
M_6	count list + approx. number + working mem. + conflict processing + resp. inhibition	5	3.77×10^{15}	3.47
M_7	count list + letter id. + approx. number + working mem. + resp. inhibition + recept. vocab.	6	3.31×10^{15}	3.96
M_8	count list + letter id. + approx. number + working mem. + conflict processing + resp. inhibition + age	7	2.70×10^{15}	4.85
M_9	count list + letter id. + approx. number + working mem. + conflict processing + age	6	2.68×10^{15}	4.88
M_{10}	count list + approx. number + working mem. + resp. inhibition	4	2.48×10^{15}	5.28

Note. count list, count list knowledge; letter id., letter–word identification; approx. number, approximate numerical comparison; working mem., working memory; conflict processing, spatial conflict processing; resp. inhibition, response inhibition; recept. vocab., receptive vocabulary. The first 10 of the 256 fitting models are presented here.

of the Bayes factors between particular models and the null model ($B_{\mathrm{m\_null}}$) indicate that these particular models are much stronger in explanatory power compared with the null model. The model with the largest value of $B_{\mathrm{m\_null}}$ was identified as the top-performing model (denoted as M_{top}). Next, we systematically computed the Bayes factors between the top-performing model and various alternative models (denoted as $B_{\mathrm{top\_m}}$; see the fifth column in Table 1) to further examine the importance of each of the variables in the top model to number knowledge. First, we compared the Bayes factor of the top model relative to other high-performing models to determine the appropriateness of the particular combination of factors included in the top model. Next, we examined the importance of each one of the variables by creating alternative models that either excluded or replaced each of the variables in the top-performing model and computed the Bayes factors between the top-performing model and these exclusion or replacement models. The underlying logic of this approach is that the Bayes factor between the top-performing model and an exclusion or replacement model indicates the difference in the explanatory power between the two models. Because this difference is due to excluding or replacing a particular variable from the top-performing model, the Bayes factor quantifies the importance of this particular variable to the top-performing model (Rouder & Morey, 2012).

To further address some more specific concerns in the existing literature on the relations among the ANS, general cognitive abilities, language abilities, and number knowledge, we also conducted an analysis dividing the approximate numerical comparison accuracy into trials demanding more executive processing (incongruent numerical and non-numerical parameters) or less executive processing (congruent numerical and non-numerical parameters) to determine whether these types of trials made similar or unique contributions to explaining individual differences in number knowledge (as seen in studies examining the relations between the ANS and general mathematics achievement, Fuhs & McNeil, 2013; Gilmore et al., 2013; Keller & Libertus, 2015).

Our analyses were implemented using Morey and Rouder's (2014) BayesFactor package (version 0.9.11–1) in R (version 3.1.3). We assessed relative importance of particular models and variables in the model by comparing Bayes factors using an accepted rule-of-thumb approach, where Bayes factors of 3 or less provide weak evidence, Bayes factors larger than 3 but less than 10 provide moderate evidence, and Bayes factors larger than 10 provide strong evidence for differences between models (Jeffreys, 1961; Wetzels et al., 2011).

Results

Variables of interest

Symbolic number knowledge was correlated with all of the target cognitive and language abilities, and many of these variables were also highly correlated (see Table 2). Descriptive statistics and correlations between variables are presented in Table 2.

Top-performing model

The eight variables of interest were entered into analyses with symbolic number knowledge as the dependent variable, and a Bayes factor was computed for each possible model relative to a model including only the intercept (M_{null}). The top-performing model (denoted as M_{top}) contained five of the eight variables: count list knowledge, letter–word identification, approximate numerical comparison ability, working memory, and spatial conflict processing (see Table 1).

Comparison of M_{top} with other models

To evaluate the appropriateness of the top-performing model (M_{top}), we first compared it with other performing models. More specifically, we computed a Bayes factor between the top-performing model and all other models, ranking the Bayes factor values in an ascending order (the smaller the Bayes factor, the closer the particular model to the top-performing model in explanatory power to data) and selecting the top 10% of all models (i.e., 24 next best models) (Moore et al., 2016; Mou et al., 2016; Rouder & Morey, 2012). Bayes factors revealed weak evidence that the top model performed better than the three next best performing models (M_2–M_4) and moderate evidence that the top model performed better than the remaining models in the top 10% of all possible models (M_5–M_{25}).

To further evaluate the appropriateness of the variables included in the top model, we counted the frequency of occurrence of each of the five variables in M_{top} in the top 10% of models. The five variables included in the top-performing model were present in a majority of the 25 models (including M_{top}). Specifically, numerical list knowledge and nonverbal approximate numerical comparison ability were present in 100% (24/24), working memory was present in 87.5% (21/24), letter–word identification was present in 75% (18/24), and spatial conflict processing was present in 66.7% (16/24) of top comparison models.

Relative importance of specific variables in the top-performing model

Comparison with exclusion models

To further determine whether each of the variables in the top-performing model was critical to characterizing early number knowledge, we computed Bayes factors between the top-performing model and next best alternative models that excluded each of the five variables in turn. If excluding a variable in the top-performing model substantially decreased the probability of observing the data, then we would conclude that the particular variable was essential to explaining variance and, thus, should be retained in a theoretical model of early number knowledge. On the other hand, if excluding a particular variable did not substantially decrease the explanatory power of the top-performing model, then we would conclude that the variable might be important but not essential to a theoretical model of early number knowledge.

Results showed that excluding any one of the five variables in the top-performing model substantially reduced the explanatory power. The Bayes factors between the top-performing model and the other four-variable models, excluding the top five variables, one by one ranged from 6.12 to 808.64,

Table 2
Descriptive statistics and correlations for the variables of interest.

	Mean (*SD*)	number knowledge	approx. number	count list	letter id.	recept. vocab	working mem.	conflict processing	resp. inhibition
Symbolic number knowledge (number knowledge)	.70 (.23)								
Approximate numerical comparison (approx. number)	.66 (.10)	**.50**							
Count list knowledge (count list)	13.78 (6.49)	**.57**	**.37**						
Letter–word identification (letter id.)	9.19 (5.10)	**.50**	**.39**	**.50**					
Receptive vocabulary (recept. vocab.)	84.31 (18.80)	**.42**	**.35**	**.32**	**.24**				
Working memory (working mem.)	.71 (.23)	**.32**	.17	.17	.17	**.38**			
Spatial conflict processing (conflict processing)	.78 (.16)	**.32**	.23	.18	.07	**.28**	.02		
Response inhibition (resp. inhibition)	.87 (.14)	**.34**	.20	**.24**	**.25**	.12	.03	**.32**	
Age	3.90 (.15)	.19	.21	.20	**.24**	**.24**	−.01	.06	.10

Note. Correlation coefficients (r) among the variables were computed with the BayesMed package (version 1.0.1) (Nuijten, Wetzels, Matzke, Dolan, & Wagenmakers, 2015) in R and are reported in the table. Bold values correspond to those correlations with a Bayes factor larger than 3, in favor of the correlation between the two variables (Wetzels & Wagenmaker, 2012). The score for symbolic number knowledge was the averaged accuracy from the give-a-number task (M = .67, SD = .28) and the what's-on-this-card task (M = .73, SD = .21) (r = .74 between the scores of the two tasks). The raw scores of letter–word identification (measured with the Woodcock–Johnson III letter–word identification subtest) and receptive vocabulary (measured with the Peabody Picture Vocabulary Test IV) are presented in the table and included in data analyses. For the convenience of comparing data across studies with the same assessments, the standardized scores of these assessments are also presented: letter–word identification (M = 110.60, SD = 12.64) and receptive vocabulary (M = 118.99, SD = 14.4).

clearly above the rule-of-thumb threshold of 3 (Jeffreys, 1961), thereby providing moderate to strong evidence that all of the variables in the top model are important for explaining individual differences in number knowledge and, thus, should be retained in a theoretical model.

Comparing Bayes factors between exclusion models (relative to the top-performing model) also provides some evidence of the relative importance of each variable. The comparison between the top-performing model and the model that excluded count list knowledge ($M_{\text{exclude\_count}}$) yielded a Bayes factor of 808.64, providing extremely strong evidence that count list knowledge was the most important variable in the top-performing model to explaining preschool symbolic number knowledge. Further comparison of the Bayes factors between the top-performing model and the other exclusion models (Table 3) suggested that after count list knowledge, approximate numerical comparison ability was next most important. Approximate numerical ability was followed by the general cognitive abilities of working memory and spatial conflict processing. Finally, knowledge of letters and words was found to be the least important of all the uniquely important variables to explaining individual differences in early number knowledge.

Comparisons with replacement models

To further understand the relative importance of the factors in the top model, we computed Bayes factors between the top-performing model and alternative models that replaced each one of the five variables in the top-performing model. If replacing a variable in the top-performing model substantially decreases the probability of the model, then this would provide more evidence that the particular variable cannot be replaced with other variables and, thus, should be retained in a theoretical model of symbolic number knowledge. On the other hand, if replacing a particular variable with

Table 3
Bayes factors for the top-performing model relative to exclusion models.

Model		Bayes factors	
Name	Variables	B_{m-null}	B_{top_m}
Top-performing model overall (M_{top})	count list + letter id. + approx. number + working mem. + conflict processing	1.31×10^{16}	1
Excluding count list ($M_{exclude\_count}$)	letter id. + approx. number + working mem. + conflict processing	1.62×10^{13}	808.64
Excluding approx. number ($M_{exclude\_approx.}$)	count list + letter id. + working mem. + conflict processing	5.44×10^{14}	24.08
Excluding working mem. ($M_{exclude\_wm}$)	count list + letter id. + approx. number + conflict processing	8.51×10^{14}	15.39
Excluding conflict processing ($M_{exclude\_conflict}$)	count list + letter id. + approx. number + working mem.	9.48×10^{14}	13.82
Excluding letter id. ($M_{exclude\_letter}$)	count list + approx. number + working mem. + conflict processing	2.14×10^{15}	6.12

Note. Each exclusion model was selected based on the next best performing four-factor model with the particular variable of interest excluded.

another does not result in a decrease of the explanatory power of the model, then we would conclude that the variable can be replaced by other variables and might not be uniquely important to a theoretical model.

Table 4 presents the alternative replacement models for each of the five variables with the next best model, including the variable that was not included in the top-performing model. Bayes factors of the top-performing model to alternative replacement models indicated that all variables in the top-performing model could not be replaced without a moderate change in explanatory power given that Bayes factors greater than 3 were observed in all cases.

Like the exclusion analysis, the replacement analysis also provided information about the relative importance of each factor in the top-performing model. Based on the Bayes factors obtained, count list knowledge appears not to be replaceable with other variables, and this suggests its unique importance to the model. Although still important, spatial conflict processing and letter–word identification were found to be less so given that the replacement models that were generated by replacing the two variables yielded only moderate to weak evidence of differing in explanatory power from the top-performing model (all with Bayes factors slightly above 3) (Jeffreys, 1961).

Additional analyses for stimulus context effects of approximate numerical comparison

As a supplement to our main analysis, we also investigated the potential role of contextual task demands in the relation between approximate numerical comparison ability and symbolic number knowledge given some reports suggesting that some types of trials are more executively demanding than others (Abreu-Mendoza et al., 2013; Fuhs & McNeil, 2013; Gilmore et al., 2013; Rousselle et al., 2004). To do this, we first computed approximate numerical comparison accuracy separately for trials where total surface area of dot arrays was positively correlated, or congruent, with numerosity and trials where total surface area was negatively correlated, or incongruent, with numerosity. We then entered approximate numerical comparison accuracy for congruent and incongruent trials simultaneously into our analyses to examine their potential contributions to explaining individual differences in symbolic number knowledge.

Children's performance on both congruent trials ($M = .74$, $SD = .13$) and incongruent trials ($M = .59$, $SD = .11$) was above chance level (chance = 50%, $ts > 9.40$, Bayes factor > 2.45×10^{13}, in favor of difference from chance). Consistent with previous studies on contextual demands on numerical comparison (Abreu-Mendoza et al., 2013; Fuhs & McNeil, 2013; Gilmore et al., 2013; Rousselle et al., 2004), children were more accurate on trials with congruent non-numerical stimulus parameters compared with trials where non-numerical parameters were incongruent with number ($t = 13.12$, Bayes factor = 2.72×10^{22}, in favor of difference between the two variables).

Table 4
Bayes factors for the top-performing model relative to replacement models.

Model		Bayes factors	
Name	Variables	$B_{\text{m-null}}$	$B_{\text{top\_m}}$
Top-performing model overall (M_{top})	count list + letter id. + approx. number + working mem. + conflict processing	1.31×10^{16}	1
Replacing count list ($M_{\text{replace\_count}}$)	**resp. inhibition** + letter id. + approx. number + working mem. + conflict processing	2.48×10^{13}	528.22
Replacing approx. number ($M_{\text{replace\_approx}}$)	count list + letter id. + **resp. inhibition** + working mem. + conflict processing	5.53×10^{14}	23.69
Replacing working mem. ($M_{\text{replace\_working. mem.}}$)	count list + letter id. + approx. number + **recept. vocab** + conflict processing	1.19×10^{15}	11.01
Replacing conflict processing ($M_{\text{replace\_conflict processing}}$)	count list + letter id. + approx. number + working mem. + **resp. inhibition**	4.16×10^{15}	3.15
Replacing letter id. ($M_{\text{replace\_letter}}$)	count list + **resp. inhibition** + approx. number + working mem. + conflict processing	3.77×10^{15}	3.47

Note. Each replacement model was selected based on the next best performing five-factor model not including the variable of interest to be replaced. The replacement variables are in bold.

Rerunning our data-driven Bayesian model selection procedure over all possible combinations of variables of interest, but with both congruent and incongruent numerical comparison accuracy entered separately (as well as all other variables as in our main analysis), resulted in the top-performing supplemental model containing five variables: count list knowledge, letter–word identification, approximate numerical comparison accuracy on the congruent trials, working memory, and spatial conflict processing. That is, the top-performing model from our supplemental analysis included the same variables as the top-performing model in our main analysis, but with approximate numerical comparison accuracy on congruent trials and not approximate number comparison accuracy on incongruent trials.

The Bayes factors between this top-performing supplementary model and all four-variable exclusion models were larger than 3 (6.3–1337.5), providing moderate to strong evidence that no variables in the top-performing model should simply be excluded. Excluding numerical comparison accuracy on the congruent trials in particular yielded a Bayes factor of 7.87, providing moderate to strong evidence for a drop in explanatory power.

All Bayes factors between the top-performing supplemental model and the replacement models for each of the five variables were also larger than 3 except for the model that replaced approximate numerical comparison on congruent trials (Bayes factor = 2.68). Importantly, the next best performing replacement model for approximate numerical comparison accuracy on congruent trials was a model that replaced approximate numerical comparison accuracy on congruent trials with approximate number comparison accuracy on incongruent trials. Thus, although performance on congruent trials was slightly more predictive, there was no more than weak evidence that two types of comparison trials differed in their explanatory power.

Discussion

In this study, we investigated the potential contribution and relative importance of various numerical, general cognitive, and language abilities to explaining individual differences at the early stages of symbolic number knowledge acquisition in preschoolers. We did so by collecting independent assessments of a variety of cognitive and language abilities hypothesized to be foundational to numerical thinking and then used a data-driven Bayesian analytic approach to systematically and objectively compare alternative models of all possible combinations of these variables to best explain individual differences in early understanding of spoken symbolic numbers. Our analyses identified five variables that uniquely contributed to explaining individual differences in preschool children's symbolic number knowledge: knowledge of the count list, nonverbal approximate numerical comparison ability, working memory, spatial conflict processing, and knowledge of letters and words. These findings are generally consistent with the predominant idea that the foundational mathematical thinking is supported by number-specific, language, and general cognitive abilities (e.g., Cirino, 2011; Dehaene et al., 2003; LeFevre et al., 2010). However, our findings move well beyond this general framework to specify more precisely the contributions (or lack thereof) of particular subcomponents within number, language and general cognition. Furthermore, our data-driven analytic approach assessing all possible combinations of variables allows us to draw novel conclusions about the relative importance of these variables to explaining individual differences in early symbolic number knowledge.

Bayes factors obtained in our study showed that preschoolers' knowledge of the count list was by far the strongest predictor of their symbolic number knowledge. A relationship between count list knowledge and symbolic number knowledge in preschoolers has also been observed in other studies (e.g., Davidson et al., 2012; Mussolin et al., 2012; vanMarle et al., 2014). In our study, count list knowledge was present in all of the best performing models (top 10% of all the alternative models), and excluding or replacing it with any other variables greatly decreased the explanatory power. Moreover, the inclusion of additional general cognitive and language variables did not diminish its contribution. Together, these results provide strong evidence that knowledge of the count list may be the most important variable to explaining individual differences in number knowledge at this age.

One may argue that the strong association between knowledge of the count list and symbolic numbers is simply due to the former being another measure of the latter given that we assessed symbolic number knowledge with tasks that involve counting. There is good reason to think that this is not an appropriate interpretation. It is well documented that count list knowledge precedes deeper number understanding given that children are able to recite the count list well before they understand what the number words mean (Fuson, 1988; Le Corre & Carey, 2007; Le Corre et al., 2006; Wynn, 1992). As evidence of this, a large majority of children in our sample knew the count list for all of the numbers we were testing (i.e., 85.5% of children correctly recited the number words over 8) despite not having a deeper understanding of their meaning. Instead, most children varied in their count list knowledge for larger numbers (up to 25) well beyond those used to test number knowledge (only to 8). Thus, although count list knowledge might be necessary, it is certainly not sufficient to grant deeper symbolic number understanding.

Regardless of whether one accepts this logic, an open question remains as to what exactly drives individual differences in count list knowledge. Differences in count list knowledge may be a proxy for experience with numbers, with greater count list knowledge reflecting more practice, exposure, or instruction on counting and numbers in general (Davidson et al., 2012). If this is the case, then experience or practice with the count list may facilitate individual differences in children's number knowledge in several respects. First, practicing the count list may enhance attention to the ordinal relations between numbers, a conceptual aspect thought to be important to a basic understanding of the symbolic number system (Brannon & Van de Walle, 2001; Le Corre et al., 2006). Second, practicing the count list may also strengthen children's representation of spoken number words and their quantitative meaning, a verbal aspect thought to be important for numerical ability (Dehaene et al., 2003; Soto-Calvo, Simmons, Willis, & Adams, 2015). Third, practicing the count list may increase children's spontaneous attention, or focusing, on number and, thus, serve as a mechanism of more continual engagement with number in children's environment (Hannula, Rasanen, & Lehtinen, 2007).

We also found that children's nonverbal approximate numerical comparison ability was related to number understanding. This finding, too, is supported by previous literature (Shusterman et al., 2016; vanMarle et al., 2014, 2016; Wagner & Johnson, 2011). Some have suggested that the ANS forms a conceptual foundation for learning the symbolic number system (Gallistel & Gelman, 1992; Gelman & Gallistel, 1978; vanMarle et al., 2016; Wagner & Johnson, 2011; but see Huntley-Fenner & Cannon, 2000, and Le Corre & Carey, 2007). Under this view, individual differences in the ANS would lead to individual differences in understanding of symbolic numbers. Others, however, have suggested and provided evidence that the relation between the ANS and symbolic number knowledge may be driven by common executive demands required in both numerical comparisons and learning numbers (Abreu-Mendoza et al., 2013; but see vanMarle et al., 2014, 2016). It is certainly the case that approximate numerical comparison involves multiple cognitive factors such as task comprehension, attentional control, and inhibition (e.g., Fuhs & McNeil, 2013; Gilmore et al., 2013; Negen & Sarnecka, 2015). However, for several reasons, our data move beyond previous work to provide strong evidence showing that at least some portion of the relation between approximate numerical abilities and number knowledge cannot be accounted for by confounding general cognitive abilities.

First, we assessed the relation between the ANS and early number knowledge after strict control for multiple components of general cognitive and language abilities that are thought to possibly confound the relationship. Second, the Bayes factor approach allowed us to not only control for general cognitive and language abilities in relation to numerical comparison but also to quantify and evaluate the relative importance of numerical comparison to other cognitive and language abilities in explaining individual differences in early number knowledge. The results showed that although both general cognitive and number-specific abilities make unique contributions to explaining individual differences, nonsymbolic approximate numerical ability appeared to be more important to early number knowledge than any of the general cognitive and language abilities. Third, extending previous work on the relations between the ANS and symbolic number understanding (Abreu-Mendoza et al., 2013; Rousselle et al., 2004; Shusterman et al., 2016; vanMarle et al., 2014, 2016), we decomposed the approximate numerical comparison accuracy in executively demanding incongruent trials and less demanding congruent trials and examined the relations between these accuracies and number knowledge after controlling for general cognitive and language abilities. We found that accuracy on congruent and incongruent trials was similarly important to explaining number knowledge after controlling for general cognitive and language abilities. This finding runs counter to the claim that executive demands of incongruent trials, not the ANS per se, drive correlations with symbolic number abilities. Here, however, we can speak only to the relation among the ANS, executive function, and early symbolic number knowledge, and our findings do not preclude the possibility that the relationships are different with later developing aspects of numerical and mathematical achievement.

Associations between general cognitive abilities and children's numerical and mathematical abilities have already been documented in previous studies (Fuhs & McNeil, 2013; Gilmore et al., 2013; vanMarle et al., 2014, 2016). Our results show that the contribution of general cognitive abilities (working memory and spatial conflict processing), while uniquely contributing, may be less important to explaining individual differences in early symbolic number knowledge than recent discussions might suggest (Gilmore et al., 2013). As evidence of this, we found that general cognitive factors (working memory and spatial conflict processing), while uniquely contributing, were much less important to explaining variability in early symbolic number knowledge than the number-related factors (count list and number comparison). We also showed that working memory and spatial conflict processing uniquely contributed to explaining individual differences in children's number knowledge, but response inhibition did not. Thus, although there is some overlap in the tasks, it may be the case that working memory and executive functions related to processing conflicting information are more important to early knowledge than executive response inhibition.

Finally, we observed evidence that the relationship between individual differences in language abilities and early symbolic number knowledge might also be weaker than some have previously thought once number-specific language (i.e., count list knowledge) is controlled (e.g., Barner, 2017; Carey, 2009; Negen & Sarnecka, 2012). Language abilities have been proposed to be a major foundation for mathematical thought, although these abilities may contribute differently to different mathematics outcomes over development (LeFevre et al., 2010). After extensively accounting for general cognitive and number-specific abilities, we did not observe that individual differences in receptive vocabulary uniquely contributed to explaining individual differences in early symbolic number knowledge. Instead, we found that individual differences in children's ability to identify letters and words made unique contributions to explaining individual differences in symbolic number knowledge. It could be that learning symbolic number does rely on verbal processing, and knowledge of letters and words reflects some aspects of individual differences in language abilities that our receptive vocabulary task does not. If this were the case, then our evidence would be consistent with the notion that language plays a unique role in early number concept development. It is also possible that the source of individual differences in children's letter–word knowledge predictive of symbolic number knowledge could be nonlinguistic in nature (Verhoeven, Reitsma, & Siegel, 2010). Regardless, it should also be noted that the contribution of letter–word identification was relatively small compared with other variables deemed uniquely important. This finding suggests that if language abilities do form a foundation for mathematical thought, then the contribution to individual differences may be smaller than that of other cognitive variables at this early point in symbolic number knowledge acquisition and smaller than it is for later developing numerical and mathematical abilities (LeFevre et al., 2010).

Conclusions

Using a Bayes factor analytic approach, our study identified the combination of variables that best explained individual differences in preschool symbolic number knowledge. These include (a) knowledge of the count list, (b) nonverbal approximate numerical ability, (c) working memory, (d) the executive ability to monitor, process, and inhibit conflicting information, and (e) the ability to identify letters and words. The variables identified as uniquely contributing are those generally expected from previous work and theorizing on symbolic number and mathematics development (e.g., Cirino, 2011; Dehaene et al., 2003; LeFevre et al., 2010). However, our findings add to existing theories by providing novel insight into the relative importance of these variables. We provide strong evidence over a variety of data-driven metrics that number-specific abilities make the largest contributions, followed by general cognitive abilities, and then knowledge of letters and words. This model suggests that there are likely many ways in which individual differences between children in number knowledge can arise. However, it also suggests that the most important source appears to be knowledge of count list (e.g., Wagner, Kimura, Cheung, & Barner, 2015), a variable that is likely to depend heavily on cultural experience and, thus, be susceptible to intervention (e.g., Berkowitz et al., 2015). It is important to note that although we draw conclusions about the potential foundations of early symbolic number thought, our data are correlational and from a single time point. As such, strong claims about directionality and causality are not warranted. Nevertheless, our model provides the most comprehensive foundation to date on which to prioritize variables in future longitudinal and experimental investigations of individual differences and intervention in early symbolic number knowledge acquisition.

Acknowledgment

This work was funded by a National Science Foundation Grant to D.C.H. (DRL 1252445).

We would like to thank Stanislas Dehaene, Manuela Piazza, and Elizabeth Spelke for their support in constructing some of our testing measures. We would also like to thank Jeff Rouder for his helpful feedback and support with the statistical analyses.

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